PDE’s on the Space of Patches for Image Denoising and Registration

David Tschumperlé* - Luc Brun*


* GREYC IMAGE (CNRS UMR 6072), Caen/France
• Definition of a Patch Space $\Gamma$.

• Patch-based Tikhonov Regularization.

• Patch-based Anisotropic Diffusion PDE’s.

• Patch-based Lucas-Kanade registration.

• Conclusions & Perspectives.
Definition of a Patch Space $\Gamma$.

- Patch-based Tikhonov Regularization.
- Patch-based Anisotropic Diffusion PDE’s.
- Patch-based Lucas-Kanade registration.
- Conclusions & Perspectives.
Located Patch of an Image

- Considering a 2D image $\mathbf{I} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^n \ (n = 3,$ for color images$)$.

- An image patch $\mathcal{P}_{(x,y)}$ is a discretized $p \times p$ neighborhood of $\mathbf{I}$, which can be ordered as a $np^2$-dimensional vector:

  $$\mathcal{P}_{(x,y)} = (I_1(x-q,y-q), \ldots, I_1(x+q,y+q), I_2(x-q,y-q), \ldots, I_n(x+q,y+q))$$

- We define a located patch as the $(np^2 + 2)$-D vector $(x, y, \lambda \mathcal{P}_{(x,y)}) \ (\lambda > 0$ balances importance of spatial/intensity features$)$. 

[Diagram: color image $\mathbf{I}$, Patch 11x11 (x3), Vector $\mathcal{P}_{(x,y)}$]
Space $\Gamma$ of Located Patches

- $\Gamma = \Omega \times \mathbb{R}^{np^2}$ defines a $(np^2 + 2)$-dimensional space of located patches.
Space \( \Gamma \) of Located Patches

- \( \Gamma = \Omega \times \mathbb{R}^{np^2} \) defines a \((np^2 + 2)\)-dimensional space of located patches.

- The Euclidean distance between two points \( p_1, p_2 \in \Gamma \) measures a spatial & intensity dissimilarity between corresponding located patches:

\[
d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + \lambda^2 SSD(P_1, P_2)}
\]

\((SSD = \text{Sum of Squared Differences})\)
We define \( \tilde{I} : \Gamma \rightarrow \mathbb{R}^{np^2+1} \), a mapping of the image \( I \) on \( \Gamma \):

\[
\forall p \in \Gamma, \quad \tilde{I}(p) = \begin{cases} 
(P^I_{(x,y)}, 1) & \text{if } p = (x, y, P^I_{(x,y)}) \\
\vec{0} & \text{elsewhere}
\end{cases}
\]
We define $\tilde{I} : \Gamma \rightarrow \mathbb{R}^{np^2+1}$, a mapping of the image $I$ on $\Gamma$:

$$\forall p \in \Gamma, \quad \tilde{I}(p) = \begin{cases} \left(\mathcal{P}_I(x,y), 1\right) & \text{if } p = (x, y, \mathcal{P}_I(x,y)) \\ \vec{0} & \text{elsewhere} \end{cases}$$

The last value of $\tilde{I}(p)$ models the meaningfulness of a located patch $p$. All patches coming from the original image $I$ have the same unit weight.

$\Rightarrow$ $\tilde{I}$ is a patch-based representation of $I$ in $\Gamma$, as an implicit surface.
Inverse Mapping to the Image Domain $\Omega$

- **Question**: Is it possible to retrieve $I$ from $\tilde{I}$?
Inverse Mapping to the Image Domain $\Omega$

**Question:** Is it possible to retrieve $I$ from $\tilde{I}$? **YES!**

$\Rightarrow$ (1) Find the most significant patches $p = (x, y, P) \in \Gamma$ for each location $(x, y) \in \Omega$:

$$p_{\text{sig}(x,y)} = \arg\max_{q \in \mathbb{R}^{np^2}} \tilde{I}_{np^2+1}(x, y, q)$$
(2) Get the central pixel of these patches, and normalize it by its meaningfulness:

\[
\forall (x, y) \in \Omega, \quad \hat{I}_i(x, y) = \frac{\tilde{I}_{ip^2+p^2+1}(x, y, \mathcal{P}_{\tilde{I}}^i(x, y))}{\tilde{I}_{np^2+1}(x, y, \mathcal{P}_{\tilde{I}}^i(x, y))}
\]

(Other solutions may be considered, for instance: averaging spatially-overlapping meaningful patches).
From Non-Local to Local processing

- Mapping $\mathbf{I}$ in $\Gamma$ transforms a non-local processing problem into a local one.

- Local or semi-local measures of $\tilde{\mathbf{I}}$ in $\Gamma$ (gradients, curvatures, ...) will be related to non-local features of the original image $\mathbf{I}$ (patch dissimilarity, variance, ...).
Main Idea of this Talk

⇒ Apply local algorithms on $\tilde{I}$ in order to build their patch-based counterparts.
⇒ Find correspondences between non-local and local algorithms.
PDE’s and variational methods are good candidates.

- They are purely local or semi-local.

- They are adaptive to local image informations (non-linear).

- They are often expressed independently on the data dimension.

- They give interesting solutions for a wide range of different (local) problems.
What Local Algorithms to Apply in $\Gamma$?

⇒ PDE’s and variational methods are good candidates.

- They are purely local or semi-local.
- They are adaptive to local image informations (non-linear).
- They are often expressed independently on the data dimension.
- They give interesting solutions for a wide range of different (local) problems.

⇒ In this talk:

- **Diffusion PDE’s** for image denoising.

- **PDE’s for image registration**, coming from a variational formulation.
• Definition of a Patch Space $\Gamma$.

⇒ **Patch-based Tikhonov Regularization.**

• **Patch-based Anisotropic Diffusion PDE’s.**

• **Patch-based Lucas-Kanade registration.**

• **Conclusions & Perspectives.**
Tikhonov Regularization in $\Gamma$ 

- We minimize the classical Tikhonov regularization functional for $\tilde{I}$ in $\Gamma$:

$$E(\tilde{I}) = \int_{\Gamma} \|\nabla \tilde{I}(p)\|^2 \, dp$$

where $\|\nabla \tilde{I}(p)\| = \sqrt{\sum_{i=1}^{np^2+1} \|\nabla I_i(p)\|^2}$
Tikhonov Regularization in $\Gamma$

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where $\|\nabla \tilde{I}(p)\| = \sqrt{\sum_{i=1}^{np^2+1} \|\nabla \tilde{I}_i(p)\|^2}$

- The Euler-Lagrange equations of $E$ give the desired minimizing flow for $\tilde{I}$:

$$\begin{cases} \tilde{I}[t=0] = \tilde{I}^{noisy} \\ \frac{\partial \tilde{I}_i}{\partial t} = \Delta \tilde{I}_i \end{cases}$$

$\Rightarrow$ **Heat flow in the high-dimensional space of patches $\Gamma$.**
This high-dimensional heat flow has an explicit solution (at time $t$):

$$\tilde{I}^{[t]} = \tilde{I}^{\text{noisy}} \ast G_{\sigma} \quad \text{with} \quad \forall p \in \Gamma, \quad G_{\sigma(p)} = \frac{1}{(2\pi \sigma^2)^{np^2+2}} e^{-\frac{||p||^2}{2\sigma^2}} \quad \text{and} \quad \sigma = \sqrt{2} t.$$
This high-dimensional heat flow has an explicit solution (at time $t$):

$$\tilde{I}^{[t]} = \tilde{I}^{noisy} \ast G_{\sigma} \quad \text{with} \quad \forall p \in \Gamma, \quad G_{\sigma(p)} = \frac{1}{(2\pi\sigma^2)^{n+2}} e^{-\frac{\|p\|^2}{2\sigma^2}} \quad \text{and} \quad \sigma = \sqrt{2t}.$$ 

Simplification: As $\tilde{I}^{noisy}$ vanishes almost everywhere (except on the original located patches of $I$), the convolution simplifies to:

$$\tilde{I}^{[t]}(x,y,P) = \int_{\Omega} \tilde{I}^{noisy}(p,q,P^{noisy}_{(p,q)}) G_{\sigma(p-x,q-y, P^{noisy}_{(p,q)}/P)} \, dp \, dq$$

⇒ Computing the solution does not require to build an explicit representation of the patch-based representation $\tilde{I}$. 

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**Solution to the Tikhonov Regularization in $\Gamma$**
Finding the most significant patches in $\Gamma$ : the flow preserves the locations of the local maxima. The inverse mapping of $\tilde{I}^t$ on $\Omega$ is then:

$$\forall (x, y) \in \Omega, \quad I^t_{(x,y)} = \frac{\int_\Omega \mathbf{I}_{(p,q)}^{noisy} w(x,y,p,q) dp \, dq}{\int_\Omega w(x,y,p,q) \, dp \, dq}$$

with $w(x,y,p,q) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-p)^2+(y-q)^2}{2\sigma^2}} \times \frac{1}{(2\pi\sigma^2)^{\frac{np^2}{2}}} e^{-\frac{\|P_{(x,y)}^{\mathbf{I}_{(p,q)}^{noisy}} - P_{(p,q)}^{\mathbf{I}_{(p,q)}^{noisy}}\|^2}{2\sigma^2}}$
Finding the most significant patches in $\Gamma$ : the flow preserves the locations of the local maxima. The inverse mapping of $\tilde{I}^{[t]}$ on $\Omega$ is then:

$$\forall (x, y) \in \Omega, \quad I^{[t]}_{(x, y)} = \frac{\int_{\Omega} I_{noisy}(p, q) w(x, y, p, q) dp \: dq}{\int_{\Omega} w(x, y, p, q) \: dp \: dq}$$

with $w(x, y, p, q) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-p)^2+(y-q)^2}{2\sigma^2}} \times \frac{1}{(2\pi\sigma^2)^2} e^{-\frac{\|P_{I_{noisy}}(x, y) - P_{I_{noisy}}(p, q)\|^2}{2\sigma^2}}$

$\Rightarrow$ Variant of the NL-means algorithm (Buades-Morel:05) with an additional weight depending on the spatial distance between patches in $\Omega$.

$\Rightarrow$ NL-means is an **isotropic diffusion process** in the space of patches $\Gamma$. 

**Inverse mapping of the Tikhonov Regularization in $\Gamma$**
Tikhonov Regularization in the Patch Space $\Gamma$

Patch-based representation $\tilde{\Gamma}$

Tikhonov Regularization

Inverse mapping in $\Omega$

Variant of the NL-Means

Original image $I$

Filtered image
(Useless) Results (Tikhonov Regularization in $\Gamma$)

Noisy color image
Tikhonov regularization in the image domain $\Omega$

($= isotropic smoothing$)
Tikhonov regularization in the $5 \times 5$ patch space $\Gamma$

($\approx$ Non Local-means algorithm)
• Definition of a Patch Space $\Gamma$.

• Patch-based Tikhonov Regularization.

⇒ Patch-based Anisotropic Diffusion PDE’s.

• Patch-based Lucas-Kanade registration.

• Conclusions & Perspectives.
Isotropic diffusion in $\Gamma$ (NL-means) does not take care of the geometry of the patch mapping $\tilde{I}$: The smoothing is done homogeneously in all directions.
- Anisotropic diffusion would adapt the smoothing kernel to the local geometry of the patch mapping $\tilde{I}$.

⇒ This anisotropic behavior can be described with diffusion tensors.
Introducing Diffusion Tensors

- A second-order tensor is a symmetric and semi-positive definite $p \times p$ matrix. ($p$ is the dimension of the considered space).

- It has $p$ positive eigenvalues $\lambda_i$ and $p$ orthogonal eigenvectors $u^{[i]}$:

$$T = \sum_i \lambda_i u^{[i]}u^{[i]T}$$
Introducing Diffusion Tensors

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\[
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\]

- Diffusion tensors describe how much pixel values locally diffuse along given orthogonal orientations, i.e. the “geometry” of the performed smoothing.
A tensor field $\mathbf{T}$ can describe locally the amplitudes and the orientations of the desired smoothing.

The smoothing itself can be performed with the application of this diffusion PDE:

$$\frac{\partial I(p)}{\partial t} = \text{trace} \left( \mathbf{T}(p) \mathbf{H}(p) \right)$$

($\mathbf{H}(p)$ is the Hessian matrix: $H_{i,j}(p) = \frac{\partial^2 I(p)}{\partial x_i \partial x_j}$)
Diffusion Tensors in Anisotropic Diffusion PDE’s

• A tensor field $T$ can describe locally the amplitudes and the orientations of the desired smoothing.

• The smoothing itself can be performed with the application of this diffusion PDE:

$$\frac{\partial I(p)}{\partial t} = \text{trace} \left( T(p) H(p) \right)$$

($H(p)$ is the Hessian matrix: $H_{i,j}(p) = \frac{\partial^2 I(p)}{\partial x_i \partial x_j}$)

⇒ How to design the tensor field $T$? ⇒ from the structure tensor field $J_\sigma$. 

Isotropic tensor field in $\Gamma$ ⇒ Isotropic smoothing

Anisotropic tensor field in $\Gamma$ ⇒ Anisotropic smoothing
The structure tensor field $\mathbf{J}_\sigma : \Omega \rightarrow \mathbb{P}(np^2 + 2)$ tells about local geometric features (local contrast, structure orientation) of $\tilde{\mathbf{I}}$:

$$\tilde{\mathbf{J}}_\sigma = \sum_{i=1}^{np^2+1} \nabla \tilde{I}_{i\sigma} \nabla \tilde{I}_{i\sigma}^T$$

where

$$\nabla \tilde{I}_{i\sigma} = \nabla \tilde{I}_i \ast G_\sigma$$

Very useful extension of the notion of “gradient” for multi-dimensional datasets. (Silvano Di-Zenzo:86, Joachim Weickert:98) used it for 2D images.

Here, we consider a $np^2 \times np^2$ structure tensor!
The structure tensor field \( \mathbf{J}_\sigma : \Omega \to \mathbb{P}(np^2 + 2) \) tells about local geometric features (local contrast, structure orientation) of \( \tilde{\mathbf{I}} \):

\[
\tilde{\mathbf{J}}_\sigma = \sum_{i=1}^{np^2+1} \nabla \tilde{I}_{i\sigma} \nabla \tilde{I}_{i\sigma}^T
\]

where \( \nabla \tilde{I}_{i\sigma} = \nabla \tilde{I}_i * G_\sigma \).

The diffusion tensor field \( \mathbf{T} \) is then designed from \( \mathbf{J}_\sigma \):

\[
\forall \mathbf{p} \in \Gamma, \quad \tilde{\mathbf{D}}_{(\mathbf{p})} = \frac{1}{\sqrt{\beta^2 + \text{trace}(\tilde{\mathbf{J}}_{\sigma(\mathbf{p})})}} \left( \mathbf{I}_d - \tilde{\mathbf{u}}_{(\mathbf{p})} \tilde{\mathbf{u}}_{(\mathbf{p})}^T \right)
\]

where \( \tilde{\mathbf{u}}_{(\mathbf{p})} \) is the main eigenvector of \( \tilde{\mathbf{J}}_{\sigma(\mathbf{p})} \).
The diffusion tensor field $T$ is then designed from $J_{\sigma}$:

$$\forall p \in \Gamma, \quad \tilde{D}_p = \frac{1}{\sqrt{\beta^2 + \text{trace}(\tilde{J}_{\sigma(p)})}} \left( I_d - \tilde{u}_p \tilde{u}_p^T \right)$$

where $\tilde{u}_p$ is the main eigenvector of $\tilde{J}_{\sigma(p)}$ (≈ normal vector to the patch-surface).
Approximation of the PDE solution

- **Problem**: Obtaining the PDE solution requires several iterations.
- But, we cannot afford to store the entire patch space $\Gamma$ in computer memory ($\text{dim}(\Gamma)=365$ for 11x11 color patches).
Approximation of the PDE solution

- **Problem**: Obtaining the solution requires several iterations.

- But, we cannot afford to store the entire patch space $\Gamma$ in computer memory ($\text{dim}(\Gamma) = 365$ for 11x11 color patches).

$\Rightarrow$ Solution of the PDE can be approximated by one iteration [Tschumperle-Deriche:03]:

$$\tilde{I}^{[t]}_{(p(x,y))} \approx \int_{(k,l) \in \Omega} I^{[t=0]}_{(k,l)} \tilde{D}_{p(x,y)} dt(p(x,y) - q(k,l)) \, dk \, dl$$

$\Rightarrow$ Solution approximation + inverse mapping on $\Omega$ can be expressed in the image domain.
Anisotropic Diffusion in the Patch Space $\Gamma$

Patch-based representation $\tilde{I}$

R(0,0) → R(1,0) → R(2,0)

Inverse mapping in $\Omega$

Anisotropic Regularization

Patch-based representation $\tilde{I}_{\text{anisotropic}}$

R(0,0) → R(1,0) → R(2,0)

Inverse mapping in $\Omega$

"Anisotropic" NL-Means

Original image $I$

Filtered image
Anisotropic Diffusion in the Patch Space (Results)
Anisotropic diffusion in the $7 \times 7$ patch space $\Gamma$
Anisotropic diffusion in the image domain $\Omega$
Anisotropic Diffusion in the Patch Space (Results)

Anisotropic diffusion in $\Omega$

Anisotropic diffusion in the patch space $\Gamma$
Anisotropic Diffusion in the Patch Space (Results)

Noisy color image
Bilateral filtering

(≈ NL-Means with 1 × 1 patches)
Anisotropic diffusion PDE in the image domain $\Omega$
Isotropic diffusion PDE in the $5 \times 5$ patch-space $\Gamma$

$(\approx$ NL-Means with $5 \times 5$ patches)$
Anisotropic diffusion PDE in the $5 \times 5$ patch-space $\Gamma$
Anisotropic Diffusion in the Patch Space (Results)

Corresponding PSNR compared to the noise-free version
• Definition of a Patch Space $\Gamma$.

• Patch-based Tikhonov Regularization.

• Patch-based Anisotropic Diffusion PDE’s.

$\Rightarrow$ Patch-based Lucas-Kanade registration.

• Conclusions & Perspectives.
The image registration problem

- Given two images $I^{t_1}$ and $I^{t_2}$, find the displacement field $u : \Omega \rightarrow \mathbb{R}^2$ from $I^{t_1}$ to $I^{t_2}$.
The image registration problem

- Given two images $I^{t_1}$ and $I^{t_2}$, find the displacement field $u : \Omega \rightarrow \mathbb{R}^2$ from $I^{t_1}$ to $I^{t_2}$

  ![Source image $I^{t_1}$](source_image.png)  ![Target image $I^{t_2}$](target_image.png)  ![Estimated displacement $u$](estimated_displacement.png)

- The Lukas-Kanade registration method is based on the minimization of:

  $E(u) = \int_{\Omega} \alpha \left\| \nabla u(p) \right\|^2 + \left\| D(p, p + u) \right\|^2 d\mathbf{p}$

- **Intensity preservation**: The intensity dissimilarity between warped $I^{t_1}$ and $I^{t_2}$ must be minimal.

  $D(p, q) = (I_{\sigma(p)}^{t_1} - I_{\sigma(q)}^{t_2})$ where $I_{\sigma}^{t_k} = I^{t_k} \ast G_{\sigma}$
We propose to solve the Lukas-Kanade problem with a dissimilarity measure defined in the patch space $\Gamma$, instead of on the image domain $\Omega$. 

Transposition to the patch-space $\Gamma$
We propose to solve the Lukas-Kanade problem with a dissimilarity measure defined in the patch space $\Gamma$, instead of on the image domain $\Omega$:

$$D_{patch}(p,q) = (\tilde{I}^t_1 \sigma(p, p_{\tilde{I}^t_1 \text{max}}(p)) - \tilde{I}^t_2 \sigma(q, p_{\tilde{I}^t_2 \text{max}}(q)))$$

i.e. Find the best 2D warp between patch representations $\tilde{I}^t_1$ and $\tilde{I}^t_2$. 

Transposition to the patch-space $\Gamma$
We propose to solve the Lukas-Kanade problem with a dissimilarity measure defined in the patch space $\Gamma$, instead of on the image domain $\Omega$:

$$D_{patch}(p,q) = \left( \tilde{I}_{t_1}^{\sigma(p,\tilde{\sigma}_{max}(p))} - \tilde{I}_{t_2}^{\sigma(q,\tilde{\sigma}_{max}(q))} \right)$$

i.e. Find the best 2D warp between patch representations $\tilde{I}_{t_1}$ and $\tilde{I}_{t_2}$.

⇒ **Patch-preservation** :
The patch dissimilarity between warped $I_{t_1}$ and $I_{t_2}$ must be minimal.

⇒ **Bloc-matching-like** dissimilarity measure + **Smoothness constraints**.
(Classical bloc-matching gives the global minimum when smoothness $\alpha = 0$).
Minimizing PDE flow

- The Euler-Lagrange equations give the minimizing flow for the patch-based Lukas-Kanade functional:

\[
\begin{align*}
\mathbf{u}_{[t=0]} &= \mathbf{0} \\
\frac{\partial u_j(x)}{\partial t} &= \alpha \Delta u_j + \sum_{i=1}^{np^2+1} \left( \tilde{I}^{t_1}_{\sigma_i(x, P_{x}^{t_1})} - \tilde{I}^{t_2}_{\sigma_i(x+u, P_{x+u}^{t_2})} \right) \nabla G_i \bigg|_{j(x+u)}
\end{align*}
\]

where \( G_{i(x)} = \tilde{I}^{t_2}_{\sigma_i(x, P_{x}^{t_2})} \).

- Local minimum of the functional.

- Resolution is done with a classical multi-scale approach (coarse to fine).
Patch-based Lukas-Kanade (Results)

Source color image
Patch-based Lukas-Kanade (Results)

Target color image
Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source

Result of the original Lukas-Kanade algorithm (smoothness $\alpha = 0.01$)
Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source

Result of the original Lukas-Kanade algorithm (smoothness $\alpha = 0.1$)
Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source

Result of the bloc-matching algorithm
(7 × 7 patches)
Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source

Result of the $7 \times 7$ Patch-Based Lukas-Kanade algorithm (smoothness $\alpha = 0$)
Patch-based Lukas-Kanade (Results)

Result of the $7 \times 7$ Patch-Based Lukas-Kanade algorithm (smoothness $\alpha = 0.01$)
• Definition of a Patch Space $\Gamma$.

• Patch-based Tikhonov Regularization.

• Patch-based Anisotropic Diffusion PDE’s.

• Patch-based Lucas-Kanade registration.

⇒ Conclusions & Perspectives.
Conclusions

(1) We proposed a patch representation $\tilde{I}$ of an image $I$ in an Euclidean patch space $\Gamma$ such that non-local operations become local ones.
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(2) We show links between local algorithms in $\Gamma$ and non-local methods in $\Omega$:

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(3) We applied more complex local methods on \( \Gamma \) to get more efficient non-local methods in \( \Omega \).

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More local methods to transpose to the patch-space $\Gamma$!

- **Texture-preserving inpainting** (Perez-Criminisi) and **Texture synthesis** (Wei-Levoy)

- Transport equations in $\Gamma$?

- You are welcome to suggest other perspectives...
• Thanks for your patience!